Quantitative Reasoning – LAB 413

t-Distributions & Inferences for Means

Week 9 Recap ▪ Lab 8

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Today's Plan

1. **Quick Recap**
   - Categorical Variables in Regression
   - t-Distributions Overview
   - t Confidence Interval
   - Matched Pairs
   - Comparing Two Means
2. **RStudio Practice**

01 Categorical Variables in Regression

- Quantitative models need numeric predictors $\rightarrow$ **dummy variables**.
- Create binary indicators for each category (omit one $\rightarrow$ avoid the "dummy variable trap").

Example

$$\text{turnout}_i = \alpha + \beta_1\,\text{Republican}_i + \beta_2\,\text{Independent}_i + \varepsilon_i$$

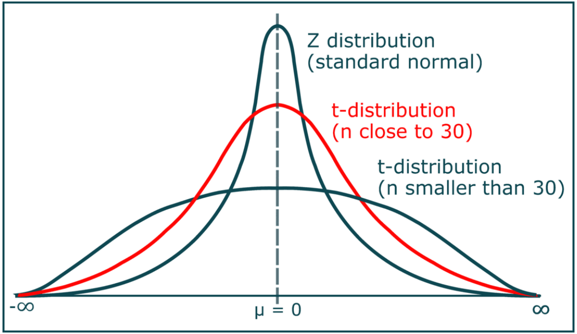
Interpretation

- $\hat{\alpha}$ = mean turnout for the reference group (e.g., Democrats).
- $\hat{\beta}$ = difference in means vs. the reference group.

Use when:

- Sample size $n < 30$
- Population σ unknown

- Replace z with t, σ with s_x.
- Degrees of freedom (df) $= n - 1$.
- t has fatter tails than Normal $\rightarrow$ more uncertainty with small n.
- As $n \rightarrow \infty$, $t \rightarrow$ Normal.



02 t-Distributions Overview (2)

Before applying t-procedures, check:

1. **Random sampling:** data should come from a random sample.
2. **Normality assumption:** is it reasonable to assume the population is roughly Normal?

Sample distribution

- If $n < 30$, check that data are roughly symmetric with no outliers.
- If $n < 15$, don't use t-procedures if data are skewed or contain outliers.
- If $n \geq 40$, t-procedures are robust even with some skew.

03 t Confidence Interval

Steps to calculate

1. Verify conditions (random sample; approx. normal data, no strong skew/outliers).
2. Identify given values.
3. Find the critical t-value t^* from the t-table (based on confidence level & df).
4. Compute the confidence interval using the formula.
5. Interpret the interval in context (e.g., "We are 95% confident that the true mean lies between ____ and ____").

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

Example: Study Hours

Scenario

A researcher wants to estimate the average number of hours Penn students study per week. She collects a simple random sample of **25 students** and finds:

- Sample mean $= 14.2$ hours
- Sample standard deviation $= 4.5$ hours

Assume the data are roughly symmetric with no clear outliers.

Construct a 95% confidence interval for the true mean study hours per week, and interpret the interval in context.

Reference: t Table

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Source: sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf

Solution: Study Hours

Given

$$
\begin{aligned}
\bar{x} &= 14.2 \\
s_x &= 4.5 \\
n &= 25 \\
df &= 24
\end{aligned}
$$

95% confidence $\rightarrow t^* = 2.064$

Compute

$$
\begin{aligned}
\text{CI} &= 14.2 \pm 2.064 \times \frac{4.5}{\sqrt{25}} \\
&= 14.2 \pm 2.064 \times 0.9 \\
&= 14.2 \pm 1.86 \\
\Rightarrow \text{CI} &= [\,12.34,\ 16.06\,]
\end{aligned}
$$

We are 95% confident that the true mean study hours per week among Penn students lies between **12.34** and **16.06** hours.

04 Matched Pairs (1)

When do we use matched pairs?

- Same subjects measured twice (before–after), or
- Two matched individuals (e.g., twins, pairs in an experiment).
- We analyze the **differences as one sample**.

Key Concepts

- **Goal:** determine whether the mean of the differences is significantly different from 0.
- If CI includes 0 $\rightarrow$ no significant difference.
- If CI excludes 0 $\rightarrow$ significant difference in means.

04 Matched Pairs (2)

Steps for a matched-pair t-test / t-interval

1. Calculate the difference for each pair: $d_i = X_{1i} - X_{2i}$
2. Compute the mean and SD of the differences.
3. Use one-sample t-procedures on d_i.
4. Build the CI using
$$\bar{d} \pm t^* \frac{s_d}{\sqrt{n}}$$
5. Interpret: if CI includes 0 $\rightarrow$ no significant difference.

Example: Online vs. In-Class Quiz

A professor wants to test whether SP2 students score differently on two types of quizzes: **online** vs. **in-class**. Five students took both versions, with scores (out of 10):

$d_i = \text{Online} - \text{In-Class}$

Student	Online	In-Class
1	8	7
2	6	6
3	9	8
4	7	7
5	10	9

Tasks

- Construct a 95% CI for the true mean difference.
- Interpret: do the two quiz formats lead to significantly different scores?

Reference: t Table

t Table

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					Confidence Level						

Source: sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf

Solution: Online vs. In-Class (1)

Step 1. Compute differences

$$d = [\,1,\ 0,\ 1,\ 0,\ 1\,]$$

Step 2. Summary statistics

$$\bar{d} = \frac{1+0+1+0+1}{5} = 0.6$$

$$d_i - \bar{d} = [\,0.4,\ -0.6,\ 0.4,\ -0.6,\ 0.4\,]$$

$$(d_i - \bar{d})^2 = [\,0.16,\ 0.36,\ 0.16,\ 0.36,\ 0.16\,] \quad \Rightarrow \quad \sum (d_i - \bar{d})^2 = 1.20$$

$$s_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{1.20}{5-1}} = \sqrt{0.30} \approx 0.548$$

Solution: Online vs. In-Class (2)

Step 3. Construct CI $(n = 5,\ df = 4,\ 95\%\text{ confidence} \to t^* = 2.776)$

$$\begin{aligned}
\text{CI} &= 0.6 \pm 2.776 \times \frac{0.548}{\sqrt{5}} \\
&= 0.6 \pm 0.68 \\
\Rightarrow \text{CI} &= [\,-0.08,\ 1.28\,]
\end{aligned}$$

Step 4. Interpretation

Since the 95% CI **includes 0**, there is no statistically significant difference between online and in-class quiz scores.

05 Comparing Two Means

- Used when there are two **independent** samples (e.g., treatment vs. control).
- Each group's observations are independent.
- **Interpretation:** the difference in sample means estimates the difference between population means (causal if randomized).
- Common in experimental or policy-evaluation settings (e.g., program participants vs. nonparticipants).

Two-sample t confidence interval

$$(\bar{x}_T - \bar{x}_C) \pm t^* \sqrt{\frac{s_T^2}{n_T} + \frac{s_C^2}{n_C}}$$

RStudio Practice

t-tests & Categorical Variables

See `Lab8.Rmd` / `Lab8_1029.R`